Trigonometry

Module 1: Trigonometric Functions

MAFS.912.F-TF.1.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle; Convert between degrees and radians. MAFS.912.F-TF.1.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. MAFS.912.F-TF.1.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. MAFS.912.G-SRT.3.7: Explain and use the relationship between the sine and cosine of complementary angles.

Dates	**Learning Targets**	**Student Evidence at Learning Target**
January 19th - February 10th	LT 1.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. (MAFS.912.F-TF.1.1) LT 1.2: Convert between degrees and radians. (MAFS.912.F-TF.1.1) LT 1.3: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise	Students will be able to extend the definition of radian measure to show that an angle measure of one radian occurs when the length of the arc and the radius of the circle are the same. (LT 1.1, LT 1.2) Students will be able to use a similarity approach to find the radian measure of central angles in circles that are not unit circles. (LT 1.1, LT 1.2) Students will be able to convert between the radian and degree measures of angles using the relationship that one revolution of the unit circle is equal to 2π radians and 360 degrees. (LT 1.2) Students will be able to draw central angles of given radian measures on the unit circle with the vertex at the origin and the initial ray on the positive x-axis. (LT 1.3) Students will be able to identify sine and cosine of an angle when given a unit circle. (LT 1.3)

	around the unit circle. (MAFS.912.F-TF.1.2) LT 1.4: Use special right triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4 and π/6. (MAFS.912.F-TF.1.3) LT 1.5: Use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. (MAFS.912.F-TF.1.3) LT 1.6: Explain and use the relationship between sine and cosine of complementary angles. (MAFS.912.G-SRT.3.7)	Students will be able to explain why co-terminal angles will all produce the same output when evaluated as inputs of a trigonometric function. (LT 1.3) Students will be able to use special right triangles (45-45-90 and 30-60-90) to find values of sine, cosine, and tangent of π/3, π/4 and π/6. (LT 1.4) Students will be able to explain that each of the triangles drawn in the first quadrant can be reflected to create a congruent triangle in the second, third, and fourth quadrant. (LT 1.4) Students will be able to use reference angles and arguments using reflections and rotations to find the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number. (LT 1.5) Students will be able to explain and use the relationship between since and cosine of complementary angles. (LT 1.6)
February 11th - 12th	**Days intended for review and module assessment.**	
Planning Resources	Questioning Planning Tool Mathematics Framework Mathematics Model Instruction Course Description	
Instructional Resources	Standards Based Assessment Items Anticipation Guide Formative Assessments Instructional Tasks	

Module 2: Graphing Trigonometric Functions and Their Inverses

<table>
<tr><td colspan="3">MAFS.912.F-TF.1.4: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

MAFS.912.F-TF.2.5: Choose trigonometric functions to model periodic phenomena with specific amplitude, frequency, and midline.

MAFS.912.F-TF.2.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</td></tr>
<tr><td>Dates</td><td>Learning Targets</td><td>Student Evidence at Learning Target</td></tr>
<tr><td>February 16th - March 4th</td><td>LT 2.1: Use the unit circle to explain symmetry (odd and even). (MAFS.912.F-TF.1.4)

LT 2.2: Use the unit circle to explain periodicity of trigonometric functions. (MAFS.912.F-TF.1.4)

LT 2.3: Choose trigonometric functions to model periodic phenomena with specific amplitude, frequency, and midline. (MAFS.912.F-TF.2.5)

LT 2.4: Understand that restricting a trigonometric functions to a domain on which it is always increasing or always decreasing allows its inverse to be created. (MAFS.912.F-TF.2.6)</td><td>Students will be able to use points and their corresponding angle measures on the unit circle to demonstrate that the cosine function is even and the sine and tangent functions are odd. (LT 2.1)

Students will be able to use arguments based on reflections to show that the cosine function is even and the sine and tangent functions are odd. (LT 2.1)

Students will be able to use the unit circle to explain that the periods for cosine and since are 2π and the period for tangent is π. (LT 2.2)

Students will be able to explain the connections between frequency and period. (LT 2.3)

Students will be able to recognize real-world situations that can be modeled with a periodic function by identifying the amplitude, frequency (or period), and midline. (LT 2.3)

Students will be able to write a function notation for the trigonometric function that models a problem situation, given the amplitude, frequency (or period), and midline of a periodic situation. (LT 2.3)</td></tr>
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		Students will be able to explain why the functions $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = \tan x$ do not have inverses using the graphs of the functions from 0 to 2π. (LT 2.4) Students will be able to restrict the domains of the graphs of functions $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = \tan x$ so that their inverses are true. (LT 2.4)
March 5th - 8th	**Days intended for review and module assessment.**	
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Module 3: Trigonometric Applications

MAFS.912.G-SRT.3.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

MAFS.912.G-SRT.4.9: Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from vertex perpendicular to the opposite side.

MAFS.912.G-SRT.4.10: Prove the Law of Sines and Cosines and use them to solve.

MAFS.912.F-TF.2.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

MAFS.912.F-TF.3.8: Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

MAFS.912.T-TF.3.9: Prove the addition and subtraction, half-angle, and double-angle formulas for sine, cosine, and tangent and use these formulas to solve problems.

Dates	**Learning Targets**	**Student Evidence at Learning Target**
March 9th - 30th	LT 3.1: Use trigonometric ratios to solve right triangles in applied problems. (MAFS.912.G-SRT.3.8) LT 3.2: Use the Pythagorean Theorem to solve right triangles in applied problems. (MAFS.912.G-SRT.3.8) LT 3.3: Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from vertex perpendicular to the opposite side. (MAFS.912.G-SRT.4.9) LT 3.4: Prove the Law of Sines. (MAFS.912.G-SRT.4.10) LT 3.5: Prove the Law of Cosines. (MAFS.912.G-SRT.4.10) LT 3.6: Use the Law of Sines and Law of Cosines to solve problems. (MAFS.912.G-SRT.4.10)	Students will be able to use angle measures to estimate the side lengths and vice versa. (LT 3.1, LT 3.2) Students will be able to solve right triangles by finding the measures of all sides and angles in the triangles using Pythagorean Theorem and/or trigonometric ratios and their inverses. (LT 3.1, LT 3.2) Students will be able to solve application problems involving right triangles, including angle of elevation and depression, navigation, and surveying. (LT 3.1, LT 3.2) Students will be able to find the length of a triangle's altitude by using the sine function. (LT 3.3) Students will be able to use the traditional area formula of a triangle $A = \frac{1}{2}(base)(height)$ and the sine function to generate an equivalent formula $A = \frac{1}{2}ab\sin C$. (LT 3.3) Students will be able to calculate the area of a triangle using the formula $A = \frac{1}{2}ab\sin C$, using any angle of the triangle. (LT 3.3) Students will be able to derive the Law of Sines by drawing an altitude in a triangle, using the sine function to find two expressions for the length of the altitude, and simplifying the equation that results from setting these expressions equal. (LT 3.4) Students will be able to derive the Law of Cosines using the Pythagorean Theorem, two right triangles formed by drawing and altitude, and substitution. (LT 3.5)

		Students will be able to generalize the Law of Cosines to apply to each included angle ($a^2 = b^2 + c^2 - 2bc\cos A$). (LT 3.5) Students will be able to use the Law of Sines and Law of Cosines to solve real world problems. (LT 3.6)
March 31st - April 16th	LT 3.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts. (MAFS.912.T-TF.2.7) LT 3.8: Evaluate the solutions of trigonometric equations using technology, and interpret them in terms of the context. (MAFS.912.T-TF.2.7) LT 3.9: Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$. (MAFS.912.T-TF.3.8) LT 3.10: Use the Pythagorean Identity to calculate trigonometric ratios. (MAFS.912.T-TF.3.8) LT 3.11: Prove the addition and subtraction formula for sine, cosine, and tangent. (MAFS.912.T-TF.3.9) LT 3.12: Prove the half-angle formula for sine, cosine, and tangent. (MAFS.912.T-TF.3.9) LT 3.13: Prove the double-angle formula for sine, cosine, and tangent. (MAFS.912.T-TF.3.9)	Students will be able to solve a trigonometric equation using an inverse trigonometric function. (LT 3.7) Students will be able to use the period of the function to identify multiple solutions of a trigonometric equation. (LT 3.9) Students will be able to evaluate solutions using a calculator. (LT 3.9) Students will be able to interpret solutions in the context of the problem. (LT 3.9) Students will be able to derive the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ by using the unit circle definitions of cosine and sine and applying the Pythagorean Theorem. (LT 3.9) Students will be able to use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to calculate the value of $\sin\theta$ or $\cos\theta$ when given $\sin\theta$ or $\cos\theta$ and the quadrant of θ. (LT 3.10) Students will be able to use right triangles to prove the sum of angles identities for cosine and sine. (LT 3.11) Students will be able to convert the sum of angles identities to the difference of angles identities. (LT 3.11)

	LT 3.14: Use the addition and subtraction, half-angle, and double angle formulas to solve problems. (MAFS.912.T-TF.3.9)	Students will be able to use the sum and difference formulas for sine and cosine to prove the sum and difference formulas for tangent. (LT 3.11) Students will be able to use the sum and difference formulas for sine, cosine, and tangent to solve for the exact values of the trigonometric functions of other angles. (LT 3.14) Students will be able to prove the half-angle formula for sine, cosine, and tangent. (LT 3.12) Students will be able to prove the double-angle formula for sine, cosine, and tangent. (LT 3.13) Students will be able to use half-angle and double angle formulas to solve problems. (LT 3.14)
April 19th - 20th	**Days intended for review and module assessment.**	
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Module 4: Vectors

MAFS.912.C-CN.2.4: Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. MAFS.912.C-CN.2.5: Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1+\sqrt{3}i)^3 = 8$ because $(-1+\sqrt{3}i)$ has modulus 2 and argument 120°.

MAFS.912.C-CN.2.6: Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

MAFS.912.N-VM.1.1: Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g. v, $|v|$, $||v||$, v).

MAFS.912.N-VM.1.2: Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

MAFS.912.N-VM.1.3: Solve problems involving velocity and other quantities that can be represented by vectors.

MAFS.912.N-VM.2.4: Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
c. Understand vector subtraction **v** – **w** as **v** + (-**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MAFS.912.N-VM.2.5: Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
b. Compute the magnitude of a scalar multiple c**v** using $||cv|| = |c|v$. Compute the direction of c**v** knowing that when $|c|v \neq 0$, the direction c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

Dates	**Learning Targets**	**Student Evidence at Learning Target**
April 21st - May 11th	LT 4.1: Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers). (MAFS.912.C-CN.2.4) LT 4.2: Explain why the rectangular and polar forms of a given complex number	Students will be able to plot complex numbers using rectangular coordinates (x, y) on the complex plane. (LT 4.1) Students will be able to plot points using polar coordinates on the coordinate plane. (LT 4.1) Students will be able to plot complex numbers using polar form (r, θ) on the complex plane. (LT 4.1)

	represent the same number. (MAFS.912.C-CN.2.4) LT 4.3: Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane. (MAFS.912.C-CN.2.5) LT 4.4: Use properties of this representation for computation. For example, $(-1+\sqrt{3}i)^3 = 8$ because $(-1+\sqrt{3}i)$ has modulus 2 and argument $120°$. (MAFS.912.C-CN.2.5) LT 4.5: Calculate the distance between numbers in the complex plane as the modulus of the difference. (MAFS.912.C-CN.2.6) LT 4.6: Calculate the midpoint of a segment as the average of the numbers at its endpoints. (MAFS.912.C-CN.2.6)	Students will be able to represent the complex point (r,θ) as the complex number $z = r\cos\theta + r\sin\theta$. (LT 4.1) Students will be able to use right triangles to derive the conversion formulas and explain why the rectangular and polar forms of a complex number represent the same number. (LT 4.2) Students will be able to add, subtract, multiply, and divide (using conjugation) complex numbers. (LT 4.3) Students will be able to calculate the modulus and argument of a complex number. (LT 4.4) Students will be able to Represent addition, subtraction, multiplication, and division of complex numbers by graphing on the complex plane. (LT 4.3) Students will be able to use geometric representation on the complex plane to perform operations with complex numbers. (LT 4.3) Students will be able to calculate the difference between two complex numbers. (LT 4.5) Students will be able to calculate the modulus of this difference, which represents the distance between the two complex numbers. (LT 4.5) Students will be able to calculate the midpoint of the line segment between two complex numbers by averaging the real and imaginary parts of the endpoints. (LT 4.6)

May 12th - June 4th	LT 4.7: Recognize vector quantities as having both magnitude and direction. (MAFS.912.N-VM.1.1) LT 4.8: Represent vector quantities by directed line segments. (MAFS.912.N-VM.1.1) LT 4.9: Use appropriate symbols for vectors and their magnitudes (e.g. v, $\|v\|$, $\|\|v\|\|$, v). (MAFS.912.N-VM.1.1) LT 4.10: Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. (MAFS.912.N-VM.1.2) LT 4.11: Solve problems involving velocity and other quantities that can be represented by vectors. (MAFS.912.N-VM.1.3) LT 4.12: Add vectors end-to-end. (MAFS.912.N-VM.2.4) LT 4.13: Add vectors component-wise. (MAFS.912.N-VM.2.4) LT 4.14: Add vectors by the parallelogram rule. (MAFS.912.N-VM.2.4)	Students will be able to explain that a vector represents two quantities, magnitude (length), and direction (angle). (LT 4.7) Students will be able to recognize that a vector can be expressed as a directed line segment, written $v = \vec{v} = \overrightarrow{PQ}$, where P is the initial point and Q is the terminal point. (LT 4.8) Students will be able to use appropriate notation for the magnitude of a vector: v, $\|v\|$, $\|\vec{v}\|$, or $\|\|v\|\|$. (LT 4.9) Students will be able to explain the component from a vector as $v = \langle v_1, v_2 \rangle = (v_1, v_2)$ in terms of horizontal and vertical travel from a starting location. (LT 4.10) Students will be able to find the component form of a vector with initial point (p_1, p_2) and terminal point (q_1, q_2) by applying the formula $v = \langle v_1, v_2 \rangle = (q_1 - p_1, q_2 - p_2)$. (LT 4.10) Students will be able to convert between component form and magnitude and direction form as appropriate. (LT 4.11) Students will be able to solve real world problems that can be represented by vectors (velocity, force, navigation, etc.) by drawing them to scale. (LT 4.12) Students will be able to solve real world problems that can be represented by vectors (velocity, force, navigation, etc.) by using vector arithmetic. (LT 4.11)

	LT 4.15: Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (MAFS.912.N-VM.2.4) LT 4.16: Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. (MAFS.912.N-VM.2.4) LT 4.17: Understand vector subtraction **v** – **w** as **v** + (-**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. (MAFS.912.N-VM.2.4) LT 4.18: Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. (MAFS.912.N-VM.2.4) LT 4.19: Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. (MAFS.912.N-VM.2.5) LT 4.20: Perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. (MAFS.912.N-VM.2.5) LT 4.21: Compute the magnitude of a scalar multiple c**v** using $\|cv\| = \|c\|v$. (MAFS.912.N-VM.2.5)	Students will be able to calculate the sum of two vectors in component form. (LT 4.13) Students will be able to add vectors by adding them end-to-end (position one vector so its initial point coincides with the terminal point of the other). (LT 4.12) Students will be able to add vectors by applying the Parallelogram Rule (position the vectors so both have their initial point at the origin). (LT 4.14) Students will be able to explain when the magnitude of the sum of two vectors does equal the sum of their magnitudes. (LT 4.15) Students will be able to explain when the magnitude of the sum of two vectors is the difference of their magnitudes. (LT 4.15) Students will be able to explain why the magnitude of the sum of two vectors does not have to equal the sum of the magnitudes of each vector $\|v + w\| \neq \|v\| + \|w\|$. (LT 4.16) Students will be able to convert from magnitude and direction form to component form to add vectors. (LT 4.16) Students will be able to convert from component form to magnitude and direction form to determine the magnitude and direction of the sum of vectors. (LT 4.16) Students will be able to explain why it is easier to calculate the sum of two vectors in magnitude and direction form by converting them to component form, adding them, and converting the components back to magnitude and direction. (LT 4.16)

	LT 4.22: Compute the direction of c**v** knowing that when $\lvert c \rvert v \neq 0$, the direction c**v** is either along **v** (for c > 0) or against **v** (for c < 0). (MAFS.912.N-VM.2.5)	Students will be able to explain that **–w** the additive inverse of **w**(**w** + **-w** = 0), has the same magnitude as **w** but points in the opposite direction. (LT 4.17) Students will be able to apply vector addition methods to solve vector subtraction problems. (LT 4.18) Students will be able to represent the scalar multiple of a vector on the coordinate plane. (LT 4.19) Students will be able to calculate the product of a scalar and a vector in the component form. (LT 4.20) Students will be able to calculate the magnitude of a scalar multiple by multiplying the absolute value of the scalar by the magnitude of **v**($\lVert cv \rVert = \lvert c \rvert \lVert v \rVert$). (LT 4.21) Students will be able to explain when scalar multiplication changes the direction of the original vector and when it does not. (LT 4.19)
June 7th - 9th	**Days intended for review and module assessment.**	
Planning Resources	Questioning Planning Tool Mathematics Framework Mathematics Model Instruction Course Description	
Instructional Resources	Standards Based Assessment Items Anticipation Guide Formative AssessmentsInstructional Tasks	